

Problem 1.45

[Difficulty: 3]

1.45 The mass flow rate of water in a tube is measured using a beaker to catch water during a timed interval. The nominal mass flow rate is 100 g/s. Assume that mass is measured using a balance with a least count of 1 g and a maximum capacity of 1 kg, and that the timer has a least count of 0.1 s. Estimate the time intervals and uncertainties in measured mass flow rate that would result from using 100, 500, and 1000 mL beakers. Would there be any advantage in using the largest beaker? Assume the tare mass of the empty 1000 mL beaker is 500 g.

Given: Nominal mass flow rate of water determined by collecting discharge (in a beaker) over a timed interval is $\dot{m} = 100 \text{ g/s}$; Scales have capacity of 1 kg, with least count of 1 g; Timer has least count of 0.1 s; Beakers with volume of 100, 500, 1000 mL are available – tare mass of 1000 mL beaker is 500 g.

Find: Estimate (a) time intervals, and (b) uncertainties, in measuring mass flow rate from using each of the three beakers.

Solution: To estimate time intervals assume beaker is filled to maximum volume in case of 100 and 500 mL beakers and to maximum allowable mass of water (500 g) in case of 1000 mL beaker.

Then
$$\dot{m} = \frac{\Delta m}{\Delta t} \quad \text{and} \quad \Delta t = \frac{\Delta m}{\dot{m}} = \frac{\rho \Delta V}{\dot{m}}$$

Tabulating results

$$\begin{array}{l} \Delta V = 100 \text{ mL} \quad 500 \text{ mL} \quad 1000 \text{ mL} \\ \Delta t = \quad 1 \text{ s} \quad \quad 5 \text{ s} \quad \quad 5 \text{ s} \end{array}$$

Apply the methodology of uncertainty analysis, Appendix E. Computing equation:

$$u_{\dot{m}} = \pm \left[\left(\frac{\Delta m}{\dot{m}} \frac{\partial \dot{m}}{\partial \Delta m} u_{\Delta m} \right)^2 + \left(\frac{\Delta t}{\dot{m}} \frac{\partial \dot{m}}{\partial \Delta t} u_{\Delta t} \right)^2 \right]^{\frac{1}{2}}$$

The uncertainties are \pm half the least counts of the measuring instruments: $\delta \Delta m = \pm 0.5 \text{ g}$ $\delta \Delta t = 0.05 \text{ s}$

$$\frac{\Delta m}{\dot{m}} \frac{\partial \dot{m}}{\partial \Delta m} = \Delta t \frac{1}{\Delta t} = 1 \quad \text{and} \quad \frac{\Delta t}{\dot{m}} \frac{\partial \dot{m}}{\partial \Delta t} = \frac{\Delta t^2}{\Delta m} \cdot -\frac{\Delta m}{\Delta t^2} = -1 \quad \therefore u_{\dot{m}} = \pm \left[u_{\Delta m}^2 + (-u_{\Delta t})^2 \right]^{\frac{1}{2}}$$

Tabulating results:

Beaker Volume ΔV (mL)	Water Collected Δm (g)	Error in Δm (g)	Uncertainty in Δm (%)	Time Interval Δt (s)	Error in Δt (s)	Uncertainty in Δt (%)	Uncertainty in \dot{m} (%)
100	100	± 0.50	± 0.50	1.0	± 0.05	± 5.0	± 5.03
500	500	± 0.50	± 0.10	5.0	± 0.05	± 1.0	± 1.0
1000	500	± 0.50	± 0.10	5.0	± 0.05	± 1.0	± 1.0

Since the scales have a capacity of 1 kg and the tare mass of the 1000 mL beaker is 500 g, there is no advantage in using the larger beaker. The uncertainty in \dot{m} could be reduced to ± 0.50 percent by using the large beaker if a scale with greater capacity the same least count were available